Pipeline leak localization using matched-field processing incorporating prior information of modeling error

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Abstract
To date, the use of matched-field processing (MFP) for leak detection in pipes has been limited to cases where the mismatch between data and model is assumed to be random and Gaussian distributed. This paper extends the MFP to the more realistic case where the mismatch involves both random and modeling errors. Experimental results show that the modeling error for cases with and without a leak remains similar in shape and magnitude. As a result, modeling errors can potentially be estimated from a baseline signal which ideally can be obtained before major defects emerge. This attribute is exploited to formulate a novel MFP technique that uses both past baseline and current signals to detect leaks. The novel MFP remains optimal in the sense of achieving maximum signal-to-noise ratio. The gain of the proposed leak detection method is assessed via three experimental scenarios in which the modeling errors range from simple to complex.

1. Introduction
Leakage in water supply system is an important problem and results in significant monetary and non-monetary losses. Since the 1990s, various transient-based leakage detection methodologies (TBLDM) have been investigated [1]. TBLDM introduces active hydraulic pressure waves [2–5], which propagate within the pipe system and thus carry information about its features. Leaksages in water pipe systems can be detected by measuring and analyzing the pressure response of the system. Specific methodological examples of this approach include (i) transient reflection-based method (TRM), such as [6–11]; (ii) transient damping-based method (TDM) by [12–14]; (iii) inverse transient analysis (ITA) method [15–18]; and (iv) frequency response-based method (FRM) by [19–30]. However, only a few articles have reported the application of these methods in real water supply pipeline systems [10,31].

Real water supply pipeline systems are often highly complex and involve a range of uncertainties, which can be classified into two categories [32]:

- Aleatoric uncertainty, also known as statistical uncertainty or random error, is representative of unknowns that differ for each experimental trial. In pipeline systems, aleatoric uncertainty is due to measurement noise, traffic noise, mechanical device error, turbulence, and air bubbles in water.

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Epistemic uncertainty, also known as systematic uncertainty or modeling error, is due to things one could, in principle, know but does not in practice. Examples of epistemic uncertainty include pipe bends [33], imprecisely known internal partial blockage or roughness, pipe material properties such as the Young’s modulus and viscoelastic parameters [34–36], and boundary conditions [37].

While the aforementioned transient-based leakage detection methods do not theoretically or analytically study the effect of aleatoric uncertainty using a probabilistic framework, the recent matched-field processing (MFP) method [34,38], estimates a leak based on an assumed probabilistic distribution of pressure measurement and the principle of maximum signal-to-noise ratio (SNR). The MFP method provides precise and robust leak estimates even in noisy environments. MFP is equivalent to the maximum likelihood (ML) approach under the Gaussian white noise assumption and can be generalized to multi-leak estimation case [39–41]. However, epistemic uncertainty, which can exceed aleatoric uncertainty, is often neglected. The neglect of epistemic uncertainty in the MFP scheme, as well as in the other published leakage detection methods, may lead to detection errors. It is precisely this shortcoming that the present paper addresses.

The influence of parameter uncertainty on transient wave propagation and leakage detection has been discussed [15,31,38,42–44]. For example, it is found that the uncertainties of wave speed and viscoelastic parameters largely affect the leak detection accuracy [44,38,34], while the uncertainty of friction factor is negligible [44]. Theoretically, these parameters can be jointly estimated along with leaks [45]. However, the joint estimation significantly increases the computational complexity and cost since a huge number of unknown parameters are typically involved. More importantly, some uncertainties cannot be modeled and parameterized. For example, “ghost fluctuations” of the measured signal have been found in transient experiments [10,31,34,46], which cannot be explained and modeled by the 1D transient wave equations [47,48]. Therefore, an alternative approach that can incorporate epistemic uncertainties in the leakage detection is desired.

In fact, epistemic uncertainty often originates from the internal properties of a piping system. Therefore, if careful monitoring is routinely practiced, prior information of epistemic uncertainty can be estimated before a leak appears. This record keeping can ideally be initiated as soon as the system is constructed, or by a regular pipe condition monitoring via transient tests. Given this ideal state of knowledge, this paper proposes a novel MFP approach which incorporates this information of epistemic uncertainty in the leakage detection procedure. The novel MFP method does not increase the computational complexity and cost, but promises a more accurate leakage estimation result.

The next section illustrates the properties of uncertainties via experimental results and then the novel MFP method is introduced. The gain of the novel MFP approach is experimentally assessed, from which conclusions are drawn.

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**Nomenclature**

- \( q \) [m\(^3\)·s\(^{-1}\)] transient discharge
- \( h \) [m] transient pressure head
- \( x^L \) [m] leak location
- \( s^L \) [m\(^2\)] leak size (effective leak area)
- \( Q_0^0 \) [m\(^3\)·s\(^{-1}\)] and \( H_0^0 \) [m] steady-state discharge and head at leak
- \( Q_0^0 \) [m\(^3\)·s\(^{-1}\)] steady-state discharge of main pipe
- \( x_m \) [m] \((m = 0, 1, \ldots, M)\) sensor coordinate
- \( h_1 \) [m] \((q_1 \) [m\(^3\)·s\(^{-1}\)]\) head (discharge) from reference (no-leak) pipe
- \( h_2 \) [m] \((q_2 \) [m\(^3\)·s\(^{-1}\)]\) head (discharge) from leaking pipe
- \( \Delta h \) [m] head difference
- \( e \) [m] random noise
- \( u \) [m] (epistemic) uncertainty
- \( \alpha \) [m·s\(^{-1}\)] wave speed
- \( A \) [m\(^2\)] internal cross-sectional area of pipe
- \( l \) [m] pipe length
- \( d \) [m] internal diameter of pipe
- \( \omega \) [Hz] angular frequency
- \( \omega_0 \) [Hz] fundamental frequency
- \( M \) sensor number
- \( J \) frequency number
- \( \text{FRF} \) frequency response function
- \( \text{HDPE} \) high-density polyethylene
- \( \text{MFP} \) matched-field processing
- \( \text{MFP-UI} \) matched-field processing with uncertainty information
- \( \text{PRV} \) pressure reducing valve
- \( \text{SNR} \) signal-to-noise ratio
2. Uncertainties in transient signal

2.1. Model

It is provisionally assumed that the reference transient data given by the monitoring of the pipe with no leak are available (Fig. 1(a)). The measured pressure head (either in time or in frequency) is represented by

\[ h_1 = h_1^{\text{mod}} + u_1 + e_1, \quad (1) \]

where \( h_1^{\text{mod}} \) is the theoretical pressure head response of the reference system, \( u_1 \) represents the epistemic uncertainty or modeling error, and \( e_1 \) denotes the aleatoric uncertainty or random noise with zero mean. Similarly, the transient response of the system with a leak (Fig. 1(b)), denoted by \( h_2 \), is also assumed as the summation of the theoretical model \( h_2^{\text{mod}}(\theta) \), an epistemic uncertainty term \( u_2(\theta) \) and a random noise term \( e_2 \):

\[ h_2 = h_2^{\text{mod}}(\theta) + u_2(\theta) + e_2, \quad (2) \]

where \( \theta \) denotes the leak parameters. In this paper, it is assumed that the epistemic uncertainty terms are invariant and independent of the leak, i.e., \( u_1 = u_2(\theta) \) def \( u \). This assumption is justified in Section 2.2. As a consequence, Eqs. (1) and (2) become

\[ h_1 = h^{\text{mod}} + u + e_1, \quad (3) \]

and

\[ h_2 = h^{\text{mod}}(\theta) + u + e_2. \quad (4) \]

2.2. Epistemic uncertainty invariance with and without leak

The invariant property of \( u \) is illustrated using three pipe systems all having different scale, geometry, boundary, pipe material and transient wave generation method, which are briefly summarized in Table 1. In these three pipe systems, the epistemic uncertainties dominate the gap between the theoretical model and the measurement of transient wave, i.e., \( e_1 \ll u_1 \) and \( e_2 \ll u_2 \), by which the invariant property of \( u \) can be more clearly observed. Within more noisy environments, \( e \ll u \) can also be realized by averaging measurements from multiple transient tests [49]. Furthermore, as indicated later in Section 3, the proposed leakage estimation method is not limited to low random noise environments.

2.2.1. Experiment I: straight copper pipe in laboratory

Experimental data in [50,51] are revisited to illustrate the property of epistemic uncertainty. Fig. 2 provides a sketch of the test rig where a straight copper pipe connects to two reservoirs. The pipe length is \( l = 37.4 \text{ m} \) and the internal diameter is

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**Fig. 1.** A schematic diagram of a piping system (a) without leak and (b) with a leak.
$d = 0.0221$ m. An initially closed valve at the downstream end generates transients by a rapid opening and subsequently closing of the valve. In this way, an approximate impulse wave is sent.

The transient test is conducted both with and without a leak. The leak is located at $x^k = 28.06$ m and the leak size (the leak effective area) is $s^l = 1.7 \times 10^{-6}$ m$^2$. Fig. 3 compares the signals for both cases measured at $x_1 = 37.4$ m. It is clear that fluctuations appear in the first half period (the signal in $[0.5, 0.55]$ s in Fig. 3). The largest fluctuation only appears in the signal from the leaking pipe and is due to the reflection from the leak. The other fluctuations are almost identical in both signal characteristics, which cannot be quantified in the theoretical transient model and their origin is unknown. In order to quantitatively evaluate the difference between the no-leak and leaking cases in terms of epistemic uncertainty, the relative error

$$
\xi = \frac{1}{R} \sum_{n=1}^{N} |h_1(t_n) - h_2(t_n)| \quad /C_0
$$

is computed, in which $h_1$ and $h_2$ respectively stand for the no-leak and leaking signals, $h_c$ is a constant head level being the steady-state head 38.4 m in this case. $t_n, n = 1, \cdots, N$, represent the selected time steps, here the first half circle (from approximately 0.5 s to 0.55 s) excluding the interval where the leak reflection appears is taken. In this case, the average absolute difference of head (the numerator of Eq. (5)) is 0.0671 m, which is much smaller than the magnitude of leak reflection being around 0.8 m. As a result, $\xi = 0.0017$. It is clear that if the reference signal is available, the leak reflection can be more easily isolated.

### 2.2.2. Experiment II: helical HDPE pipe in laboratory

The experimental setup is shown in Fig. 4. A helical high-density polyethylene (HDPE) pipe with $l = 188.89$ m, $d = 0.0933$ m and wall thickness $e = 0.0081$ m is used. The upstream is connected to a tank and the downstream to a valve (for transient wave generation). The steady-state discharge at the downstream is $Q_0 = 1.95 \times 10^{-3}$ m$^3$ s$^{-1}$. In Fig. 5, the results from three transient tests performed in this pipe system are shown: without leak, with one leak at $x^{k1} = 110.53$ m, and with two leaks at $x^{k1} = 110.53$ m and $x^{k2} = 83.19$ m. The effective sizes of the two leaks are respectively $s^{k1} = 3.36 \times 10^{-3}$ m$^2$ and $s^{k2} = 6 \times 10^{-5}$ m$^2$. The date of the first two tests is 12 October 2018, while the third test (two leaks) was performed four days later. Fig. 5 compares the measured pressure head at $x^{k1} = 110.53$ m for the three cases. It is clear that the three signals have almost identical features of epistemic uncertainty, for example, for $t \in [4.9, 5.3]$ s, fluctuations with same shape consistently appear. However, unlike the experiment in Section 2.2.1 where the only difference between the no-leak and leaking cases is leak signature, more uncertainties exist in this pipe system. First, the Joukowsky head (the maximum pressure) in the two-leak case is slightly lower than the other two cases. This is because, for the given supply pressure, the two-leak condition results in a higher loss of downstream pressure as well as a smaller value of the downstream pressure, the two-leak condition results in a higher loss of downstream pressure as well as a smaller value of the downstream pressure. Second, the shape of signal in approximately $t \in [5.6, 5.9]$ s has slight difference, due to the multiple reflections from the two leaks. However, the epistemic uncertainty in this system with and without leaks is still approximately invariant. The relative error in Eq. (5) is $\xi = 0.0053$ in the single-leak case and $\xi = 0.0135$ in the two-leak case, where $t_n$'s are taken from the time instant at which the Joukowsky head is reached to the first leak reflection (approximately from 4.9 s to 5.3 s) and $h_c$ is the Joukowsky head in this case.

### Table 1: Experiments considered in this paper.

<table>
<thead>
<tr>
<th>I</th>
<th>II</th>
<th>III</th>
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</thead>
<tbody>
<tr>
<td><strong>Location</strong></td>
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<td>Laboratory</td>
</tr>
<tr>
<td><strong>Upstream boundary</strong></td>
<td>Reservoir</td>
<td>Tank</td>
</tr>
<tr>
<td><strong>Source signal</strong></td>
<td>Impulse</td>
<td>Step (valve closure)</td>
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<tr>
<td><strong>Pipe material</strong></td>
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<tr>
<td><strong>Pipe geometry</strong></td>
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<td>Helical</td>
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<tr>
<td><strong>Pipe length</strong></td>
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<td>188.89 m</td>
</tr>
<tr>
<td><strong>Internal diameter</strong></td>
<td>0.0221 m</td>
<td>0.0933 m</td>
</tr>
</tbody>
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![Fig. 2. Setup of the pipe system in Experiment I.](image-url)
2.2.3. Experiment III: non-straight HDPE pipe with an upstream PRV (Beacon Hill, Hong Kong)

The experimental setup is shown in Fig. 6. Water flows from an upstream reservoir on a mountain top to a pressure reducing valve (PRV) via a long transmission main. The test section extends from the PRV to a downstream ball valve which generates transients through a fast closure (see Fig. 6(b)). The length of the HDPE pipe in the test section is $l = 135.84$ m.

Fig. 3. Pressure head time signal in Experiment I from the intact pipe (no leak) and the leaking pipe at $x_1 = 37.4$ m.

Fig. 4. Setup of the experiments in the Water Engineering Laboratory at University of Perugia, Italy.

2.2.3. Experiment III: non-straight HDPE pipe with an upstream PRV (Beacon Hill, Hong Kong)

The experimental setup is shown in Fig. 6. Water flows from an upstream reservoir on a mountain top to a pressure reducing valve (PRV) via a long transmission main. The test section extends from the PRV to a downstream ball valve which generates transients through a fast closure (see Fig. 6(b)). The length of the HDPE pipe in the test section is $l = 135.84$ m, with
internal diameter $d = 0.147 \text{ m}$ and pipe wall thickness $e = 0.0164 \text{ m}$. The pipe has a U-shape with a $90^\circ$ elbow. The initial steady-state discharge at the downstream is approximately $Q_0 = 5 \times 10^{-3} \text{ m}^3 \cdot \text{s}^{-1}$.

Fig. 7(a) shows the measurements at $x_1 = 133.34 \text{ m}$ from two experiments without leak and two experiments with leak; the experiment dates of the two no-leak tests and the two leaking tests were roughly three weeks apart (20 October 2018 to 8 November 2018). The PRV automatically adjusts its opening to regulate the outlet pressure to a preset value [52–54]. In the no-leak case, as soon as the PRV senses the transient pressure increase, it fully closes. As a result, pressures do not return to their initial values. In the case with leak, as there is still flow after valve closure, the PRV does not fully close. The downstream pressure rapidly decreases due to the water release and local head loss across the PRV. A more detailed analysis of the physical mechanism of PRV for this pipe system can be found in [36]. The uncertainties in this experimental scenario are clearly greater and more aleatoric than in Experiments I and II. The Joukowsky head is not identical and the local fluctuation is not coincident in the different tests. However, the significant fluctuations due to uncertainties for the case with and without leak have similar shape. This can be more clearly observed through the averaged signals shown in Fig. 7(b); the corresponding relative error in Eq. (5) is $\zeta = 0.0153$ where $h_i$ is the Joukowsky head and the time steps from the Joukowsky head to the first leak reflection are selected. This implies that assumption of invariant epistemic uncertainty, i.e., $u_1 = u_2$, still approximately holds. Since the fluctuations due to uncertainties are even more significant than the signature of leak, quantifying the influence of these uncertainties would significantly improve the leakage detection accuracy.

3. Leak localization methodology

3.1. Frequency-domain pressure head model

The transient model with and without leak ($h_1^{\text{mod}}$ and $h_2^{\text{mod}}$ in Eqs. (1) and (2)) is now explicitly given. Due to the property introduced in Section 2.2, the epistemic uncertainty term is identical in both cases, i.e., $u_1 = u_2 = u$. The frequency-domain pressure head (frequency response function (FRF)) in the cases without and with leak is

$$ h_1 = h_N^L(\tilde{q}_1(x^{U})) + u + e_1 $$

and

$$ h_2 = h_N^L(\tilde{q}_2(x^{U})) + s^T G(\tilde{q}_2(x^{U})) + u + e_2. $$

Here, $h_i$ ($i = 1, 2$) is an $M \times J$-dimensional vector where $M$ is the number of sensors and $J$ is the number of frequencies, $\tilde{q}_i(x^{U})$ is the estimate of the discharge at the pipe upstream end $x^{U}, i = 1, 2$. The derivation of $\tilde{q}_i(x^{U})$ and the details of $h_N^L$ and $G$ are given in Appendix A.

3.2. Matched-field processing without consideration of epistemic uncertainty

The MFP method in [34,38] (without consideration of epistemic uncertainty) assumes $u = 0$ and only uses $h_2$ to detect a leak. The leak location is estimated by best matching the data $\Delta h_2 = h_2 - h_N^L(\tilde{q}_2)$ and its theoretical model, which is realized by maximizing the inner product between $\Delta h_2$ and (normalized) $G(x^{U})$.
\[
\hat{x}^2 = \arg \max_{\hat{x}^2} |B|^2 = \arg \max_{\hat{x}^2} \left< \Delta h_2, \frac{g'(\hat{x})}{|g'(\hat{x})|} \right>^2 \\
= \arg \max_{\hat{x}^2} \Delta h_2 \frac{g'(\hat{x})}{\|g'(\hat{x})\|} \Delta h_2
\]

where \(\cdot, \cdot\) stands for the inner product and the superscript \(H\) represents the conjugate transpose of a vector. As soon as the leak location is estimated, the leak size can be estimated by the classical least squares [38]:

**Fig. 6.** Setup of the pipe transient experiment in Beacon Hill, Hong Kong.
If $u = 0$, the objective function in Eq. (8) is
\[ |B|^2 = \left(s^H(\chi^l) + e^H(\chi^l) \right) G(\chi^l) (s^H(\chi^l) + e^H(\chi^l)) \].

In this case, the leak estimate, decided by the maximum of Eq. (10), is optimal in the sense of achieving maximum signal-to-noise ratio (SNR) \cite{38}. However, in reality, since $u$ is nonzero, the objective function plotted for leakage localization becomes
\[ |B|^2 = \left(s^H(\chi^l) + e^H(\chi^l) \right) G(\chi^l) (s^H(\chi^l) + e^H(\chi^l)) .
\]

It is clear that $u$ distorts the expected objective function Eq. (10); this may change the shape and the maximum of the objective function and, thus, inaccurately locate the leak.

3.3. Novel matched-field processing: incorporating prior information of uncertainty

A novel MFP approach is now proposed, which utilizes the measurement $h_1$ from a previous test with no leak. With the measurement $h_1$, the epistemic uncertainty term $u$ could be estimated by
\[ \hat{u} = h_1 - h^{\text{NL}}(q^l(\chi^{l'})). \]

It can be found from Eq. (6) that $\hat{u}$ is an unbiased estimate of $u$, i.e., $E(\hat{u}) = u$, but has the 0-mean random error $e_1$ ($\hat{u} = u + e_1$). Considering this identification of $u$ in the model of $h_2$ in Eq. (7), we obtain

\[ s^H = G^H(\chi^l) h_2 . \]
\[ \mathbf{h}_2 = \mathbf{h}^{\text{NL}}(\mathbf{q}_2(x^U)) + s^I \mathbf{G}(x^L; \mathbf{q}_2(x^U)) + \mathbf{h}_1 - \mathbf{h}^{\text{NL}}(\mathbf{q}_1(x^U)) - \mathbf{e}_1 + \mathbf{e}_2. \] (13)

Let
\[ \Delta \mathbf{h}_{12} \overset{\text{def}}{=} \Delta \mathbf{h}_2 - \Delta \mathbf{h}_1 = \mathbf{h}_2 - \mathbf{h}^{\text{NL}}(\mathbf{q}_2(x^U)) - (\mathbf{h}_1 - \mathbf{h}^{\text{NL}}(\mathbf{q}_1(x^U))) \] (14)

be the data for leakage detection (\( \mathbf{h}_1 \) and \( \mathbf{h}_2 \) are the measured pressure heads from the \( M \) sensors; \( \mathbf{h}^{\text{NL}}(\mathbf{q}_1(x^U)) \) and \( \mathbf{h}^{\text{NL}}(\mathbf{q}_2(x^U)) \) are computed from Eqs. (A.7) and (A.1) which also use the pressure head measurements from an additional sensor) and it has the theoretical expression
\[ \Delta \mathbf{h}_{12} = s^I \mathbf{G}(x^L) + \mathbf{e}_{12}. \] (15)

Here, \( \mathbf{e}_{12} = \mathbf{e}_2 - \mathbf{e}_1 \) is still a zero-mean random variable with variance \( \sigma_1^2 + \sigma_2^2 \), where \( \sigma_1^2 \) and \( \sigma_2^2 \) are respectively the variances of \( \mathbf{e}_1 \) and \( \mathbf{e}_2 \). Therefore, based on the principle of MFP [38], the leak location and size can be estimated:
\[ \hat{x} = \arg \max_{x^L} |B| = \arg \max_{x^L} \frac{\Delta \mathbf{h}_{12}^{\text{NL}} \mathbf{G}(x^L) \mathbf{h}^{\text{NL}}(x^L) \Delta \mathbf{h}_{12}}{\mathbf{G}^I(x^L) \mathbf{G}(x^L)} \] (16)

and
\[ \hat{s}^I = \frac{\mathbf{G}^I(\hat{x}) \Delta \mathbf{h}_{12}}{\mathbf{G}(\hat{x}) \mathbf{G}(\hat{x})}. \] (17)

Note that Eqs. (8) and (16) imply that the novel MFP does not change the structure of the solution (thus no additional computational cost is incurred), but rather modifies the input data via Eq. (14). This automatically accomplishes the identification of uncertainty and incorporates this knowledge into the leakage detection scheme. The novel MFP also remain optimal in dealing with random noise by maximum SNR.

4. Experimental results

In this section, the novel MFP approach is tested via the experimental data from the three scenarios introduced in Section 2.2.

4.1. Experiment I

The experimental setup can be found in Fig. 2. Pressures are measured by three hydrophones at \( x_0 = 6.7 \) m, \( x_1 = 37.4 \) m, and \( x_2 = 18.2 \) m in the cases with and without leak: the time signals and the corresponding FRFs are shown in Figs. 8 and 9, respectively. With the FRFs at the three hydrophones in Fig. 9, the leak localization is realized via MFP. The pressure head measurement from \( x_0 \) is used to derive \( \mathbf{q}_1 \) and \( \mathbf{q}_2 \) (cf. Appendix A for details), while the measurements from \( x_1 \) and \( x_2 \) compose \( \Delta \mathbf{h}_{12} \), i.e., \( M = 2 \). The used frequencies are \( \{\omega : \omega / \omega_{1b} \in (0, 21)\} \), where \( \omega_{1b} = \pi / (2l) = 55.8 \) rad/s is the fundamental frequency and the wave speed \( a = 1328 \) m/s, i.e., all the frequencies lower than the 11th resonant peak.

Fig. 10(a) shows the result of the MFP without prior information of uncertainty [51], which plots the objective function in Eq. (8). It is clear that the highest peak (at 29.28 m) is close to the actual leak location (at 28.06 m; the dash line in Fig. 10) but has an error of 1.22 m. This error arises because the uncertainty (for example, the local fluctuations in Fig. 3) distorts the objective function Eq. (8), as indicated by Eq. (11). By contrast, as is shown in Fig. 10(b), the novel MFP with uncertainty information (abbreviated as MFP-UI), which plots Eq. (16), is much more accurate that the leak location estimate is \( \hat{x} = 28.08 \) m (the error is 0.02 m). Note that the plot in Fig. 10(b) has a simulation-like shape (cf. Fig. 4 in [38] as an example). This is because the uncertainties in the intact and leaking cases are almost identical (cf. Fig. 3), which are effectively canceled by MFP-UI. Note that there exists a secondary peak in Fig. 10(b) which is symmetric with respect to the actual leak (at \( x = 8.8 \) m; approximately \( l - x^c \), a feature arising from the symmetric property of \( \mathbf{G}(x^L) \) [38]. However, there are several methods that can easily rule out the possibility of this secondary peak as a leak, for example, plotting Fig. 10(b) using different hydrophones [38], the phase method [22], and the criterion of energy loss at resonant frequency [55]. As this topic is outside of the scope of the present paper, it is not detailed here.

The novel MFP-UI method also largely improves the leak size estimation. The leak size estimate is \( \hat{s} = 6.1 \times 10^{-6} \) m² from the MFP without prior information of uncertainty and \( \hat{s} = 1.3 \times 10^{-6} \) m² from the MFP-UI, while the actual leak size is \( s = 1.7 \times 10^{-6} \) m².
4.2. Experiment II

The experiments in the Water Engineering Laboratory at University of Perugia, Italy, are now considered. The experimental setup is shown in Fig. 4. The upstream of the pipe is connected to a tank and, thus, \( h(x^0) = 0 \) is assumed. The transient wave is generated by the sudden valve closure at \( x_D \). Three hydrophones at \( x_0 = 7.92 \text{ m}, x_1 = 187.66 \text{ m}, \) and \( x_2 = 111.88 \text{ m} \) are used to measure pressures. Three transient tests are performed: without leak, with one leak at \( x_{L1} = 110.53 \text{ m}, \) and with two leaks at \( x_{L1} = 110.53 \text{ m} \) and \( x_{L2} = 83.19 \text{ m}. \) The measured pressure heads in time and in frequency (FRF) from the three hydrophones are shown in Figs. 11 and 12. The FRFs at frequencies \( \{\omega: \omega/\omega_h \in (0, 14)\} \) are used for leakage localization with MFP. The viscoelastic model is used here; the parameters used in the Kelvin-Voigt viscoelastic model are [34]: Poisson ratio \( \nu = 0.43, \tau_1 = 0.05 \text{ s}, J_1 = 0.951 \times 10^{-10} \text{ Pa}^{-1}, \tau_2 = 0.5 \text{ s}, J_2 = 1.065 \times 10^{-10} \text{ Pa}^{-1}, \tau_3 = 1.5 \text{ s}, J_3 = 0.815 \times 10^{-10} \text{ Pa}^{-1}. \)

First, the single leakage detection is considered; the results from the MFP without prior information of uncertainty and MFP-UI are shown in Fig. 13. It is clear that the MFP without consideration of uncertainties cannot accurately and uniquely localize the leak: although there is a local maximum of the MFP objective function Eq. (8) close to the actual leak, there are other local maxima and it increases sharply and significantly near both ends of pipe. On the other hand, the proposed MFP-UI method is able to eliminate these interferences and has a unique maximum at \( \hat{x} = 104 \text{ m} \) (the actual leak location is \( x_L = 110.53 \text{ m} \)). The leak localization error is 6.5 m which is higher than Section 4.1 (0.02 m); this likely arises because of unknown system changes that cannot be fully canceled via MFP-UI. However, the error is reasonable as it is much lower than the minimum wavelength \( \lambda_{\text{min}} = 54 \text{ m} \). Furthermore, the leak size estimate is \( \hat{s} = 1.4 \times 10^{-4} \text{ m}^2 \) from the MFP without prior information of uncertainty and \( \hat{s} = 1.05 \times 10^{-4} \text{ m}^2 \) from the MFP-UI, while the actual leak size is \( s = 0.6 \times 10^{-4} \text{ m}^2 \). This implies that both MFP methods overestimate the leak size due to the presence of uncertainties [41], but MFP-UI more approaches the actual value.

Fig. 14 displays the localization of two leaks at \( x_{L1} = 110.53 \text{ m} \) and \( x_{L2} = 83.19 \text{ m}. \) Again, the proposed MFP-UI method achieves a better result that the highest two maxima of the objective function correspond to the two actual leaks, while...
Fig. 9. FRF measurements in Experiment I in (a) no-leak pipe and (b) leaking pipe (the leak is located at $x_L = 28.06$ m). The measurement locations are $x_0 = 6.7$ m, $x_1 = 37.4$ m, and $x_2 = 18.2$ m.

Fig. 10. Leakage localization in Experiment I using (a) MFP without prior information of uncertainty and (b) MFP-UI. The actual leak location $x^*$ is at 28.06 m; the estimate $\hat{x}$ is 29.28 m (the error is 1.22 m) in (a) and 28.08 m (the error is 0.02 m) in (b).
the MFP without prior information of uncertainty fails to find both leaks. Note that even though both MFP methods here use a model with single leak, the methods can roughly estimate multiple leaks via the 1D search of leak location (Eqs. (8) or (16)) if the leaks are not too close to each other (not lower than half minimum wavelength). More analysis of single-leak model for multi-leak localization using MFP can be found in [38]. Moreover, a more accurate and robust estimation of multiple leaks should consider multi-leak model (cf. [39–41]), which has more computational complexity and requires more computational cost. Finally, note that in the two-leak estimation in Fig. 14, the data \( h_1 \) and \( h_2 \) are not obtained on the same day, which confirms the validity of the assumption of invariant epistemic uncertainties in the sense of successful leakage detection using MFP-UI.

### 4.3. Experiment III

Finally, the experimental results from the Leakage Detection and Pressure Management Training Field in Beacon Hill, Hong Kong, are presented. The experimental setup is shown in Fig. 6. A ball valve is set at \( x_{U} \) to generate transient wave by valve closure. The viscoelastic coefficients of the HDPE pipe material in the Kelvin-Voigt model are: the Poisson ratio \( v = 0.45, \tau_1 = 0.05 \text{ s}, \tau_2 = 0.5 \text{ s}, \tau_3 = 1.5 \text{ s}, J_1 = 1.667 \times 10^{-10} \text{ Pa}^{-1}, J_2 = 0.102 \times 10^{-10} \text{ Pa}^{-1}, J_3 = 1.010 \times 10^{-10} \text{ Pa}^{-1} \). Four

![Fig. 11. Time domain measurements of pressure head in Experiment II in the pipe (a) with no leak, (b) with one leak at \( x^*_L = 110.53 \text{ m} \), and (c) with two leaks at \( x^*_L = 110.53 \text{ m} \) and \( x^*_L = 83.19 \text{ m} \). The measurement locations are \( x_0 = 7.92 \text{ m}, x_1 = 187.66 \text{ m}, \) and \( x_2 = 111.88 \text{ m} \).](image-url)
pressure transducers are set in the pipe: one is at just downstream of the PRV, whose location is denoted as $x_{U} + 0$, to measure $h(x_{U} + 0/C_{0}/C_{1})$; the other three hydrophones are located at $x_{0} = 40.04 \, \text{m}$, $x_{1} = 88.48 \, \text{m}$, and $x_{2} = 133.34 \, \text{m}$. The measured pressure heads in time in the cases of intact and leaking pipe are displayed in Fig. 15. Fig. 16 shows the magnitude of FRF at all the four hydrophones, in the both intact and leaking cases. Note that in the leaking case, there is an additional peak in FRF at around 0.25 Hz. This is because in the corresponding time signal (cf. Fig. 15(b)), the tail of signal forms sine-like function whose period is approximately 4 s.

Unlike the previous cases where the boundary condition $h(x_{U}) = 0$ is assumed, in this experiment $h(x_{U})$ is dynamic as a PRV is located at $x_{U}$, thus it is replaced by the measurement $h(x_{U} + 0)$. Fig. 17 shows the leakage localization results using (a) MFP without prior information of uncertainty and (b) MFP-UI, where the elastic model is used (the wave speed is assumed as in Eq. (A.6)). For estimating $q(x_{U})$ (cf. Eq. (A.7)), it is assumed that there is no leak between $x_{U} + 0 \, \text{m}$ and $x_{0} = 40.04 \, \text{m}$ (cf. Appendix A). In this experiment, another pipe is connected to the upstream of $x_{U}$, such that the boundary condition of $q(x_{U})$ can also be estimated via measurements in that pipe. Then, the assumption of no-leak between $x_{U} + 0$ and $x_{0}$ is not necessary. For this reason, the MFP objective function between $x_{U} + 0$ and $x_{0}$ is plotted but using dotted line. It is clear from Fig. 17(a) that the leak localization using the MFP without prior information of uncertainty is inaccurate: the estimated leak location (the peak of the solid line) is $x_{L} = 76.83 \, \text{m}$ while the actual leak is at $x_{L} = 76.83 \, \text{m}$ and another (higher) peak appears at around 30 m. On the other hand, as is indicated by Fig. 17(b), the proposed MFP-UI approach localizes the leak at $x_{L} = 78.4 \, \text{m}$ (an
error of 1.57 m) and there is no other significant peak. Moreover, the estimated leak size using the MFP without prior information of uncertainty and the proposed MFP-UI is respectively $b_sL = 10.14/10^{-4} m^2$ and $b_sL = 1.41/10^{-4} m^2$; the actual leak size is approximately $s_L = 0.7/10^{-4} m^2$. Again, due to the large amount of uncertainty, the MFP without prior information of uncertainty largely overestimates the leak size, while the estimate from MFP-UI is much closer to the actual value as the epistemic uncertainties have been effectively canceled in MFP-UI.

Fig. 18 displays the leakage localization results as Fig. 17 but with the viscoelastic model, i.e., the wave speed in Eq. (A.5) is used. In this case, the leak localization is $\hat{x}^b = 78.04 m$ (the error is 1.21 m) using MFP-UI and is $\hat{x}^b = 68.04 m$ (the error is 8.79 m) using MFP without prior information of uncertainty: in the both cases, including viscoelasticity in the model improves the result.
5. Conclusions

A novel matched-field processing (MFP) method is proposed which considers the epistemic uncertainties in the leakage estimation scheme. The uncertainty information is obtained from a prior transient test with negligible leakage. This method is based on the assumption that these uncertainties are time-invariant and independent of appearance of leak. This assumption is demonstrated as reasonable at least in three experimental scenarios, ranging from simple to complex in terms of the complexity of uncertainty. In all the three cases, the novel approach of MFP with uncertainty information (MFP-UI) improves the leakage detection result than the previous version of MFP.

It is well-advised to perform a transient test after a pipe system is constructed, or, even better, to routinely conduct transient tests. Using these measurements as a reference and combining with the MFP-UI method have the potential to greatly enhance the accuracy of transient-based leakage detection. Application of the proposed methodology in more complicated real-life pipe networks would be an interesting and logical next step.

CRediT authorship contribution statement

Xun Wang: Conceptualization, Methodology, Software, Data collection, Writing - original draft, Writing - review & editing. Muhammad Waqar: Conceptualization, Methodology, Software, Data collection, Writing - original draft, Writing - review & editing. Hao-Chen Yan: Data collection. Moez Louati: Writing - review & editing. Mohamed S. Ghidaoui: Conceptualization, Methodology, Writing - review & editing. Pedro J. Lee: Data collection, Writing - review & editing. Silvia Meniconi: Data collection, Writing - review & editing. Bruno Brunone: Data collection, Writing - review & editing. Bryan Karney: Writing - review & editing.
**Fig. 16.** FRF in Experiment III with (a) intact pipe and (b) leaking pipe (the leak locates at $x^l = 76.83$ m). The measurement locations are $x^{U} = 0$ m, $x_0 = 40.04$ m, $x_1 = 133.34$ m, and $x_2 = 88.48$ m.

**Fig. 17.** Leak localization in Experiment III using (a) MFP without prior information of uncertainty and (b) MFP-UI. The actual leak location $x^l = 76.83$ m. The elastic transient model is used.
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Transient model in the frequency domain

In Eqs. (6) and (7), \( h_i (i = 1, 2) \) is a vector composed of pressure heads from \( M \) sensors \( (x_1, \cdots, x_M) \) and \( J \) frequencies \( (\omega_1, \cdots, \omega_J) \), denoted by \( h_i = \text{vec} \{ h_i(x_m, \omega_j) : m = 1, \cdots, M, j = 1, \cdots, J \} \). Similarly, \( e_i = \text{vec} \{ e_i(x_m, \omega_j) : m = 1, \cdots, M, j = 1, \cdots, J \} \) and \( u = \text{vec} \{ u(x_m, \omega_j) : m = 1, \cdots, M, j = 1, \cdots, J \} \) represents the noise and uncertainty vectors, respectively. Moreover,

\[
\begin{align*}
\mathbf{h}_{\text{NL}}(\mathbf{q}(\mathbf{x}^U)) &= \text{vec} \{ h_i(\mathbf{q}(\mathbf{x}^U), x_m, \omega_j) : m = 1, \cdots, M, j = 1, \cdots, J \},
\end{align*}
\]

(A.1)

in which

\[
\begin{align*}
h_i(\mathbf{q}(\mathbf{x}^U), x_m, \omega_j) &= -Z_j \sinh (\beta_j x_m) q_i(\mathbf{x}^U, \omega_j) + \cosh (\beta_j x_m) h_i(\mathbf{x}^U, \omega_j) .
\end{align*}
\]

(A.2)

Here,

\[
Z_j = \beta_j a_r^2(\omega_j)/(i\omega_g A)
\]

(A.3)

is the characteristic impedance, \( g \) is the gravitational acceleration, \( A \) is the cross-sectional area,

\[
\beta_j = a_r^{-1}(\omega_j) \sqrt{-\omega_j^2 + igA\omega_j R}
\]

(A.4)
is the propagation function, $R$ is the frictional resistance term and $R = (f_{cw} Q_0) / (gd^2)$ for turbulent flows, $f_{cw}$ is the Darcy–Weisbach friction factor, $Q_0$ is the steady-state discharge in the pipe,

$$a_{re}(\omega) = \left( \rho \left( \frac{1}{R} + (1 - v^2) \frac{d}{e^2} J_0 + \sum_{j=1}^{N_0} \frac{J_1}{1 + \mathrm{i} \omega \tau_j} \right) \right)^{\frac{1}{2}}$$

(A.5)

is the frequency-dependent wave speed due to the pipe wall viscoelastic effect [34,56-60] (Experiments II and III in this paper). In Eq. (A.5), $\rho$ and $\kappa$ are the density and bulk modulus of water, $v$ is the Poisson ratio of pipe wall, $e$ is the pipe thickness, $J_0 = 1/E$ where $E$ is the Young’s modulus of the pipe, $N_0$ is the truncated order of the generalized Kelvin-Voigt model [61], $J_1$ and $\tau_j$ are the creep coefficients. Here, the coefficients $J_1$ and $\tau_j$ can be estimated from a transient signal before leak detection, even unknown leaks or other discrete reflectors exist in the pipe [62]. Note that if it is an elastic pipe (Experiment I in this paper), the viscoelastic effect, i.e., the summation term in Eq. (A.5), is neglected, thus the wave speed becomes

$$a_e = \left( \rho \left( \frac{1}{R} + (1 - v^2) \frac{d}{e^2} J_0 \right) \right)^{\frac{1}{2}}.$$

(A.6)

The effect of unsteady friction is neglected in the transient model because the model considering the pipe wall viscoelasticity but without considering the unsteady friction can satisfactorily mimic the transient pressure wave attenuation, dispersion, and shape [63–65].

In Eq. (A.2), the pressure head at the upstream boundary $h_i(x^U, \omega_j)$ is assumed (Experiments I and II) or measured (Experiment III). The discharge $q_i(x^U, \omega_j)$ is estimated by setting a sensor at $x_0$ which is close to $x^U$ and there is no leak between $x^U$ and $x_0$. More exactly, given the measurement $h_i(x_0, \omega_j)$ at $x_0$, the discharge at $x^U$ is estimated [66,67,39] by

$$\hat{q}_i(x^U, \omega_j) = \frac{\cosh(\beta_j(x_0 - x^U)) h_i(x^U, \omega_j) - h_i(x_0, \omega_j)}{Z_j \sinh(\beta_j(x_0 - x^U))}.$$  

(A.7)

When a leak exists in the pipe, $z^L, Q^L$, and $H^L_0$ denote the elevation of the pipe at the leak, the steady-state discharge and head at the leak, respectively. The lumped leak parameter $s^L = C^L A^L$ stands for the effective leak size, where $C^L$ is the discharge coefficient of the leak and $A^L$ is the flow area of the leak opening (orifice). The steady-state discharge of the leak is related to the lumped leak parameter by $Q^L_0 = s^L \sqrt{2g (H^L_0 - z^L)}$. In Eq. (7),

$$G(x^L; \hat{q}_2(x^U, \omega_j), x_m, \omega_j)$$

where

$$G(x^L; \hat{q}_2(x^U, \omega_j), x_m, \omega_j) = - \frac{\sqrt{Z_j \sinh(\beta_j(x_m - x^L))}}{\sqrt{2g (H^L_0 - z^L)}} \left( Z_j \sinh(\beta_j x^U) \hat{q}_2(x^U, \omega_j) - \cosh(\beta_j x^U) \hat{q}_2(x^U, \omega_j) \right).$$

(A.9)

References


